Extraction of skewed parton distributions.

Dieter Müller

Institut für Theoretische Physik, Universität Regensburg,
D-93040 Regensburg, Germany

Skewed parton distributions contain new non-perturbative information about hadronic states. Thus, their extraction from experimental data is an important goal. Properties and models for skewed parton distributions as well as their extraction, based on perturbative leading-order results, are shortly reviewed.

Talk given at the 8th International Workshop on Deep Inelastic scattering and QCD, Liverpool, April 25-30, 2000.

1 Definition and properties of Skewed parton distributions.

Skewed parton distributions (SPD's) were introduced for about one decade as a generalization of both Feynman's parton densities as well as of exclusive distribution amplitudes [1]. They are defined as expectation values of light-ray operators sandwiched between hadronic states $|P_i S_i\rangle$ with different momentum P_i and spin S_i dependence. At twist-two level they read in the quark sector:

$$\left\{ \begin{array}{c} {}^{Q}q^{V} \\ {}^{Q}q^{A} \end{array} \right\} (x, \xi, \Delta^2, \mu) = \int \frac{d\kappa}{2\pi} e^{i\kappa x P_+} \langle P_2 S_2 | \bar{\psi}(-\kappa n) \left\{ \begin{array}{c} \gamma_+ \\ \gamma_+ \gamma_5 \end{array} \right\} \psi(\kappa n) | P_1 S_1 \rangle_\mu, \quad (1)$$

where x is the longitudinally momentum fraction with respect to $P_+ = n(P_1 + P_2)$ [n is a light-cone vector which project onto the $+$ component], $\xi = -\Delta_+/P_+$ is the so-called skewedness parameter with $\Delta = P_2 - P_1$ and μ is the renormalization scale of the operators. They describe the probability amplitude to find a quark with momentum fraction $(x-\xi)P_+/2$ which goes for the DGLAP–region, i.e. $|x| \geq |\xi|$, into a final quark with momentum fraction $(x+\xi)P_+/2$ or forms together with the second quark a mesonic like state into the ER-BL–region, i.e. $|x| \leq |\xi|$.

More recently, it has been realized that they contain valuable non-perturbative information, which, for instance, may offer the possibility to measure the angular momentum fraction of quarks and gluons in the nucleon. A number of properties follow from first principals by means of the definition (1):

- Support properties: $q(x, \xi) = 0$ for $|x| > 1$.
- Polynomial property of moments: $\int_{-1}^{1} dx x^j q(x, \xi, \Delta^2) = \sum_{k=0}^{j} q_{jk}(\Delta^2)\xi^k$.
- Lowest moment is given by form factors: $q_{00} = \langle P_2 S_2 | n^\mu {}^{Q}J^I_\mu | P_1 S_1 \rangle / P_+$.
- Hermiticity as well as time reversal invariance provide: $q(x, \xi) = q(x, -\xi)$.
- Evolution equation follows from renormalization group invariance: $\mu^2 \frac{d}{d\mu^2} q(x, \xi, \Delta^2, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|} V(x/\xi, y/\xi; \alpha_s(\mu)) q(y, \xi, \Delta^2, \mu)$.
- Inclusive connection: $q(x, \mu^2) = q(x, \xi = 0, \Delta^2 = 0, \mu^2)_{|S_2 \to S_1}$, where $q(x, \mu^2)$ is the usual parton density.
- Exclusive connection: $\Phi(x, \mu^2) = q(x, \xi = -1, \Delta^2 = M^2, \mu^2)_{\langle P_2, S_2| \to \langle 0|}$, where $|P_1, S_1\rangle$ is now a meson state and Φ is the distribution amplitude.

2 Models for skewed parton distributions.

As we see from the list given above, the SPD's give us a link between exclusive quantities like electromagnetic form factors and parton densities measured in inclusive reactions. Based on model assumptions, there exist different proposals for the SPD's with quite different characteristics. For instance, the bag-model predicts a valence quark distribution which is rather independent on the skewedness parameter [2]. While the chiral solution model in the large N_c limit takes into account the Dirac sea and, thus, predicts a more complex shape of SPD's containing zeros and a strong skewedness dependence [3]. Note that the typical sclale for these predictions is very low, i.e. $Q_0 \approx 0.4$ GeV and $Q_0 \approx 0.6$ GeV, respectively.

Other suggestions are inspired by the inclusive connection and are based on different mappings of the parton densities to the SPD's, for instance, equating them, mapping the Mellin moments to the conformal ones due to an integral transformation, or expressing the SPD's in terms of double distributions [4]. Thereby, it is assumed that the Δ^2 dependence is factorized, which is certainly true up to logarithmic corrections for large Δ^2 of a few GeV. Note that in the forward limit the spin flip part vanishes, and therefore, further assumptions are needed. For very small values of ξ, the non-spin flip part in the DGLAP region is essentially determined by the forward parton distributions.

These prescriptions have to be supplemented by the scale at which they are applied. It seems to be a good idea to use a low input scale that is typically for non-perturbative model calculations. Afterwards one evolves the SPD's to the scale that is used in hard scattering experiments, i.e. higher than at least one GeV. The advantage is twice: i) one study the stability under evolution and ii) perturbative QCD information are taken into account. Note that evolution to an asymptotic large scale predicts, independently on the initial conditions, that the SPD's are concentrated in the ER-BL region and vanish for $x = \pm\xi$. However, this fact is not of practical relevance for the scales accessible in experiments.

3 Experiments to access skewed parton distributions.

There is a growing interest to access these non-perturbative functions in lepton-hadron scattering experiments, where the virtuality of the intermediate photon has to be larger than of at least one GeV. On the theoretical side there are formal factorization proofs for the following hard processes to leading twist-two accuracy in a perturbative calculable hard-scattering part and universal SPD's: deeply virtual Compton scattering (DVCS) $e^{\pm}p \to e^{\pm}p\gamma$, exclusive

meson M production $e^\pm p \to e^\pm B M^a$, and exclusive lepton pair production $e^\pm p \to e^\pm p l^+ l^-$. In the first two cases the amplitude reads in leading order:

$$\mathcal{A}(\xi,\Delta^2,Q^2) \propto \sum_{j=u,d,s} \int_{-1}^{1} dx \left(\frac{C_j}{x-\xi+i\epsilon} + \frac{\overline{C}_j}{x+\xi-i\epsilon} \right) q_j(x,\xi,\Delta^2,Q^2), \quad (2)$$

where the coefficients $C_j, \overline{C}_j$ depend on the considered process. If $\mathcal{A}$ could be very precisely measured as function of ξ and Q^2, the deconvolution of this formula exists in principal. Practically, one has to compare the model predictions with the experimental data or defines characteristic functions that together with eq. (2) can distinguish between different models:

$$R = \frac{\mathrm{Re}\mathcal{A}(\xi,\Delta^2,Q^2)}{\mathrm{Im}\mathcal{A}(\xi,\Delta^2,Q^2)}, \quad S = 1 - \frac{\mathrm{PV}\int_{-1}^{1} dx \frac{1}{x-\xi} \mathrm{Im}\mathcal{A}(x,\Delta^2,Q^2)}{(-\pi)\mathrm{Re}\mathcal{A}(\xi,\Delta^2,Q^2)}. \quad (3)$$

The latter one can be considered as a measure for the skewedness dependence.

Unfortunately, this perturbative leading order analysis can be spoiled by the size of perturbative as well as higher twist corrections. The first ones have been considered for DVCS in the valence quark region at next-to-leading order. It turns out that the corrections due to evolution are small, i.e. about 10% or less, while the corrections to the hard scattering amplitude depend on the chosen model and can be of the order of 50% or even more [5].

1. F.-M. Dittes and et al., Phys. Lett. B209:325, 1988; D. Müller and et al., Fortschr. Phys. 42:101, 1994; X. Ji, Phys. Rev. D55:7114, 1997; A.V. Radyushkin, Phys. Lett. B380:417, 1996.
2. X. Ji, W. Melnitchouk, and X. Song, Phys. Rev. D56:5511, 1997.
3. V.Yu. Petrov and et al., Phys. Rev. D57:4325, 1998.
4. A. Shuvaev, Phys. Rev. D60:116005, 1999; A.V. Radyushkin, Phys. Rev. D56:5524, 1997.
5. A.V. Belitsky and et al., Phys. Lett. B437:160, 1998; Nucl. Phys. B546:279, 1999; Phys. Lett. B474:163, 2000.

[a] Since B denotes an arbitrary baryon, the SPD's defined in eq. (1) are generalized to "off-diagonal" ones. Note also that factorization is only proofed for longitudinal polarized photon.